4 –. Solve the simultaneous differential equations using Laplace Transform

With the initial conditions $x\ (0) = 35; \dot{x}(0) = -48, y(0) = 27, and \dot{x}(0) = -55$

$$\frac{d^2x}{dt^2} + \frac{dy}{dt} + 3x = 15e^{-t}$$

$$\frac{d^2y}{dt^2} + -4\frac{dx}{dt} + 3y = 15\sin(2t).$$

Plot the responses $x(t)$ and $y(t)$

$x(10) = 35, x(0) = -48, y(0) = 27, y(0) = -55$

* $s^2x(8) - sx(0) - \dot{x}(0) -+ sy(s) - y(0) + 3x(x) = \frac{15}{s+1}$

$(s^2+3)x(s) + sy(s) = \frac{15}{s+1} + 35s - 21$

$s^2y(s) - sy(0) - y(0) - 4\{sx(s) - s(0)\} + 3y(s) =$

$(s^2+3)y(s) - 4sx(y) = \frac{30}{s^4+4} + 27s - 125$

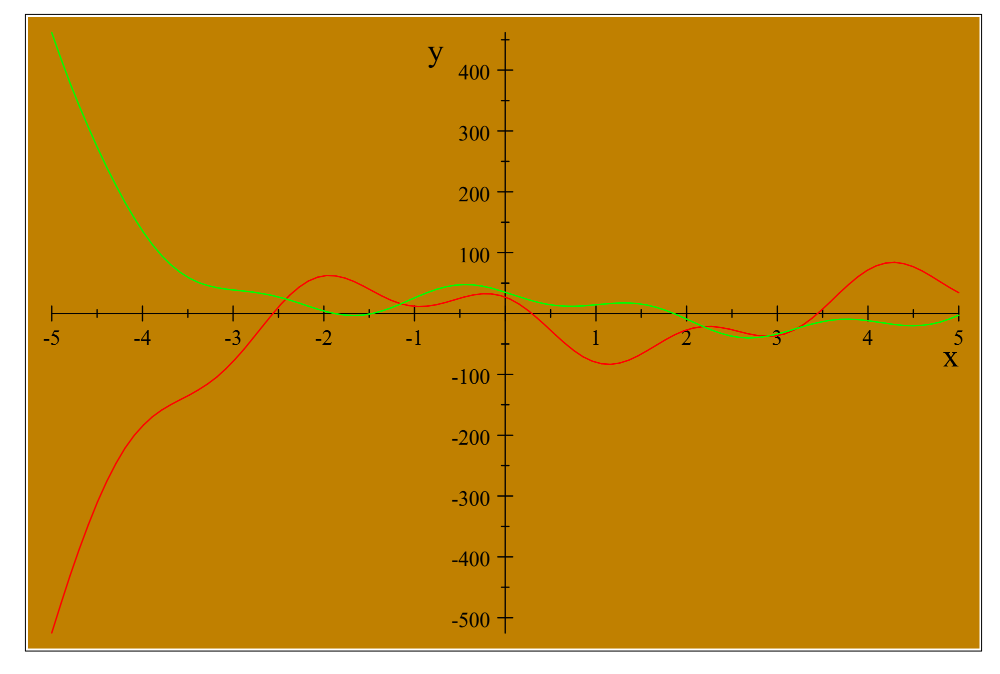
So, $y(s) = \frac{\left[4s\left(\frac{15}{s+1} + 35s - 21\right) + s^2 + \frac{30}{s^2+4} + 27s - 125)\right]}{[4s^2 + (s^2+3)^2]}$

$x(s) = \frac{1}{s^2+3}\frac{15}{s+1} + 35s - 21 - s\frac{\left[4s\left(\frac{15}{s+1} + 35s - 21\right) + s^2 + \frac{30}{s^2+4} + 27s - 125)\right]}{[4s^2 + (s^2+3)^2]}$

Inverse Laplace Tranform, We get

$s(t) = -15\sin(3t) + 30\cos(t) + 2\cos(2t) + 3e^{-t}$

$y(t) = -60\sin(t) + \sin(2t) + 30\cos(3t) - 3e^{-t}$



5.- A periodic function of period 10 is defined within the period $-5 < t < 5$ by

$f(t) = \{ {}^{0 \quad (-5<t<0)}_{3 \quad (0<t<5)}$

Determine its Fourier series expansion and illustrate graphically for $-12 < t < 12$.

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nwt) + \sum_{n=1}^{\infty} b_n \sin(nwt)$$

$$a_n = \frac{2}{T}\int_d^{d+T} f(t)\cos(nwt)dt(n = 0,1,2)$$
$$b_n = \frac{2}{T}\int_d^{d+T} f(t)\sin(nwt)dt(n = 0,1,2,3)$$
$$T = \frac{2 * \pi}{\omega}$$
$$T = 10$$

so, $\omega = \frac{2 \cdot \pi}{t} = \frac{2 \cdot \pi}{10} = \frac{1}{5}\pi$ or $\frac{5}{\pi}$

$$a_0 = \frac{2}{10} \cdot \left[\int_{-5}^{0} 0 + dt + \int_{-5}^{0} 3 \cdot dt\right]$$
$$= \frac{1}{5} \cdot 3 \cdot t|_0^5$$
$$= 3$$
$$a_n = \frac{2}{10} \cdot \left[\int_{-5}^{0} 0 + \cos\left(\frac{n\pi t}{5}\right)dt + \int_0^5 3 \cdot \cos\left(\frac{n\pi t}{5}\right)dt\right]$$
$$= \frac{1}{5} \cdot 3\int_0^5 \cos\left(\frac{n\pi t}{5}\right)dt$$

$$= \left[\frac{3}{5} \cdot \frac{5}{n \cdot \pi} \cdot \sin\left(\frac{n\pi t}{5}\right)|_0^5\right]$$
$$= \frac{3}{n \cdot \pi} \cdot [1 - \cos(n\pi)]$$
$$= \frac{3}{n \cdot \pi} \cdot [1 - (-1)^n] = \{ {}^{\frac{6}{n \cdot \pi},\ n\ odd}_{0,\ \ n\ even}$$

The fourier expasion is

$$f(t) = \frac{3}{2} + \sum_{n\ odd} \frac{6}{n \cdot \pi} \cdot \sin(\frac{n\pi t}{5}) =$$

$$\frac{3}{2} + \frac{6}{\pi}\sum_{n=1}^{\infty}\left\{\frac{1}{2n-1}\sin\left[\frac{(2n-1\pi t)}{5}\right]\right\} : \frac{6}{\pi}\sum_{n=1}^{\infty} \frac{\sin\left(\frac{2}{5}n - \frac{1}{5}\pi t\right)}{2n-1} + \frac{3}{2}$$

6-. Use the Gram-Schmidt orthogonalization process to transform the given basis
$B = \{u1, u2, u3\} for R3$
into an orthogonal basis $B\prime = \{v1, v2, v3\}$. Then form an orthonormal basis $B" = \{w1, w2, w3\}$.

$$B = \{[-3,1,1],[1,1,0],[-1,4,1]\}$$
$$v_1 = u_2\left(\frac{u_2 \cdot v_1}{v_1.v_1}\right)v1$$
$$= \langle 1,1,0\rangle - \frac{\langle 1,1,0\rangle \cdot \langle -3,1,1\rangle}{\langle -3,1,1\rangle \cdot \langle -3,1,1\rangle}\langle -3,1,1\rangle$$
$$= \langle 1,1,0\rangle - \left(\frac{-3+1+0}{9+1+1}\right)\langle -3,1,1\rangle := -\frac{2}{11}\langle -3,1,1\rangle$$
$$= \langle 1,1,0\rangle - \left\langle \frac{6}{11}\frac{2}{11}\frac{2}{11}\right\rangle = \frac{5}{11}, \frac{13}{11}, \frac{2}{11}$$

$$v_3 = u_3 - \left(\frac{u_3 \cdot v_1}{v_1.v_1}\right)v_1 - \left(\frac{u_3 \cdot v_2}{v_2.v_2}\right)v_2$$

$$= \langle 1,4,1\rangle - \left\langle -\frac{24}{11}\frac{8}{11}\frac{8}{11}\right\rangle - \left\langle \frac{245}{198}, \frac{637}{198}, \frac{49}{99}\right\rangle$$

$$= \left\langle -\frac{1}{18}\frac{1}{18}\frac{2}{9}\right\rangle$$

There, $\acute{B} = \left\{\langle -3,1,1\rangle, \left\langle \frac{5}{11}\frac{13}{11}\frac{2}{11}\right\rangle - \left\langle \frac{1}{18}\frac{1}{18}\frac{2}{19}\right\rangle\right\}$

$$w_1 = \frac{v_1}{\|v_1\|}$$

$$= \frac{-3,1,1}{\sqrt{-3^2 1^2 + 1^2}} = \frac{-3,1,1}{\sqrt{11}} = -\frac{3}{\sqrt{11}}\frac{1}{\sqrt{11}}\frac{1}{\sqrt{11}}$$

$$w_1 = \frac{v_2}{\|v_2\|}$$

$$= \frac{\left\langle \frac{5}{11}\frac{13}{11}\frac{2}{11}\right\rangle}{\sqrt{\left(\frac{5}{11}\right)^2 + \left(\frac{13}{11}\right)^2 + \left(\frac{2}{11}\right)^2}}$$

$$= \frac{\left\langle \frac{5}{11}\frac{13}{11}\frac{2}{11}\right\rangle}{\sqrt{\frac{25}{121}\frac{169}{121}\frac{4}{121}}} := \frac{\left\langle \frac{5}{11}\frac{13}{11}\frac{2}{11}\right\rangle}{\sqrt{\frac{18}{11}}} =$$

$$= \left\langle \sqrt{\frac{18}{11}}\frac{5}{11}\frac{\sqrt{11}}{11}\frac{13}{\sqrt{18}} \cdot \frac{\sqrt{11}}{11}\frac{2}{\sqrt{11}} \cdot \frac{\sqrt{11}}{11}\right\rangle :$$

$$= \frac{5}{3\sqrt{2}} \cdot \frac{1}{\sqrt{11}}\frac{13}{3\sqrt{2}} \cdot \frac{1}{\sqrt{11}}\frac{2}{3\sqrt{2}} \cdot \frac{1}{\sqrt{11}}$$

$$= \frac{5}{3\sqrt{22}}\frac{13}{3\sqrt{22}}\frac{2}{3\sqrt{22}}$$

$$w_1 = \frac{v_3}{\|v_3\|}$$

$$= -\frac{1}{3\sqrt{2}}\frac{1}{3\sqrt{2}} - \frac{4}{3\sqrt{2}}$$

There, $\ddot{B}$ $\left\{\left\langle -\frac{3}{\sqrt{11}}\frac{1}{\sqrt{11}}\frac{1}{\sqrt{11}}\right\rangle \left\langle \frac{5}{3\sqrt{22}}\frac{13}{3\sqrt{22}}\frac{2}{3\sqrt{22}}\right\rangle \left\langle -\frac{1}{3\sqrt{2}}\frac{1}{3\sqrt{2}} - \frac{4}{3\sqrt{2}}\right\rangle\right\}$

Result

$\acute{B} = \left\{\langle -3,1,1\rangle, \left\langle \frac{5}{11}\frac{13}{11}\frac{2}{11}\right\rangle - \left\langle \frac{1}{18}\frac{1}{18}\frac{2}{19}\right\rangle\right\}$

$\ddot{B}$ $\left\{\left\langle -\frac{3}{\sqrt{11}}\frac{1}{\sqrt{11}}\frac{1}{\sqrt{11}}\right\rangle \left\langle \frac{5}{3\sqrt{22}}\frac{13}{3\sqrt{22}}\frac{2}{3\sqrt{22}}\right\rangle \left\langle -\frac{1}{3\sqrt{2}}\frac{1}{3\sqrt{2}} - \frac{4}{3\sqrt{2}}\right\rangle\right\}$

7. –Using the Lyapunov approach investigate the stability of the system described by the

state
equation

$$A = \begin{bmatrix} -4 & 2 \\ 3 & -2 \end{bmatrix}$$

Take Q to be the unit matrix. Confirm your answer by determining the eigenvalues of the state matrix.

$$-\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} + \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}\begin{bmatrix} -4 & 2 \\ 3 & -2 \end{bmatrix}$$

$$= -\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -4p_{11} + 3p_{21} & -4p_{12} + 3p_{22} \\ 2p_{11} - 2p_{21} & 2p_{12} - 2p_{22} \end{bmatrix} + \begin{bmatrix} -4p_{11} + 3p_{12} & -2p_{11} - 2p_{12} \\ -4p_{21} + 3p_{22} & 2p_{21} - 2p_{22} \end{bmatrix} =$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 3p_{12} - 8p_{11} + 3p_{21} & 3p_{22} - 6p_{12} - 2p_{11} \\ 2p_{11} - 6p_{21} + 3p_{22} & 2p_{12} + 2p_{21} - 4p_{22} \end{bmatrix} =$$

$3p_{12} - 8p_{11} + 3p_{21} = -1$

$2p_{11} - 6p_{21} + 3p_{22} = 0 \models\Longrightarrow\Longrightarrow p_{11}\frac{5}{8}, p_{12} = \frac{2}{3}$

$3p_{22} - 6p_{12} - 2p_{11} = 0$

$2p_{12} + 2p_{21} - 4p_{22} = -1$

$p21\frac{2}{3}, p_{22} = \frac{11}{12}$

$$p = \begin{bmatrix} \frac{5}{8} & \frac{2}{3} \\ \frac{2}{3} & \frac{11}{12} \end{bmatrix}$$

P are and det $P\left|\frac{5}{8}\right|0$ and $\det P = \frac{55}{96} - \frac{4}{9} = \frac{111}{864} > 0$

Reult

$v(x) = \frac{5}{8}x^2 + \frac{4}{3}x_1x_2 + \frac{11}{12}x_2^2$